Lesson 32 - Lagrange Multipliers II
Applications

Last class, we learned how to use Lagrange Multipliers to find extrema (maxima and minima) of a function of two variables on a curve.

maximize (or minimize):	$z = f(x, y)$
subject to:	$\mathrm{g}(x, y) = 0$

Today we will use this test to help us solve optimization word problems.

I Strategy for Lagrange Multipliers Optimization Problems

(1) **Read** the problem. Then **read it again** for details.

(2) Write your **variables** and what they mean.

(3) Write an **objective function** and whether you are trying to **maximize** it or **minimize** it.

(4) Write the **constraint equation** specified in the problem. (This is the **subject to** part of the problem.)

(5) Use the **Lagrange Multipliers** to find places where the maximum or minimum may occur.

(6) Test another point on the constraint equation to ensure that you have found a maximum or minimum.

(7) **Answer the question.** Read carefully to know if they want the point where the extremum occurs or if they want the extreme value (plug point into $f(x, y)$.)

II Examples

Example 1 (Problem 23 in Section 7.5 of *Applied Calculus*, 9th edition, by Hoffman and Bradley)**.** A manufacturer has \$8,000 to spend on the development and promotion of a new product. It is estimated that if x thousand dollars is spent on development and y thousand dollars is spent on promotion, sales will be approximately

$$f(x, y) = 50x^{1/2}y^{3/2}$$

units. How much money should the manufacturer allocate to development and how much to promotion to maximize sales?

Example 2 (Based on a problem on a LON-CAPA problem)**.** The temperature at a point (x, y) on a plate is given by

$$T(x, y) = 3x^2 + 2y^2 - 18x + 16y$$

degrees Celsius. An ant travels in a circle on the plate that has center at $(3, -4)$ and radius 5. What is the hottest temperature encountered by the ant? The coldest temperature?

Example 3 (Problem 35 in Section 7.5 of *Applied Calculus*, 9th edition, by Hoffman and Bradley). A rectangular storage shed with a flat roof is to be constructed of material that costs \$15 per square foot for the roof, \$12 per square foot for the two sides and back, and \$20 per square foot for the front. Suppose that you wanted to find the dimensions of the shed of largest volume that could be constructed for \$8000. Write down the system of equations that you would need to solve if you were using Lagrange multipliers for this problem. **DO NOT SOLVE the system.**